EGARCH VERSUS PARCH APPROACH IN MODELING DEVELOPED AND UNDERDEVELOPED STOCK MARKETS

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Abstract
Based on recent results regarding the changes in volatility modeled by the instrumentality of different specifications of Generalized Autoregressive Conditionally Heteroscedastic (GARCH) models, the aim of this paper is to make an analysis of volatility through comparison. Thus, we take into account two asymmetric models from the GARCH family (the Exponential Generalized Autoregressive Conditionally Heteroscedastic Model - EGARCH and the Power Autoregressive Conditionally Heteroscedastic Model - PARCH) under the assumption of the two most used distributions of the innovations (Gaussian and Student's t) in two stock markets at opposite poles: London Stock Exchange represented by FTSE Index (developed stock market) and Bulgarian Stock Exchange represented by SOFIX Index (underdeveloped stock market). The interesting point is that the PARCH(1,1) with asymmetric order 1 and Student's t distribution performs better than all the EGARCH models in estimating the conditional variance in case of the FTSE 100 Index, thus the developed market is characterized by the leverage effect, but in case of the Bulgarian Stock Exchange neither EGARCH nor PARCH are good models for estimating the volatility of the market, a fact that leads to many questions, including why the leverage effect is not present in this market.

Keywords: Conditional Variance, Leverage Effect, EGARCH, PARCH, International financial markets, FTSE 100 Index, SOFIX Index

JEL Classification: C22, C52, C55, C58

Introduction

In their recent research paper Petrică, et al. (2016) examine the changes in volatility in case of the Tokyo Stock Exchange through the NIKKEI 225 and TOPIX Indices by taking into account three asymmetric GARCH models: EGARCH, TARCH and PARCH, estimated using the maximum likelihood method under the assumption of five distributions of the error terms*. They find that EGARCH model performs better than TARCH and PARCH models. Petrică and Stancu (2017a) also study the volatility of the Romanian stock market where they employ both symmetric and asymmetric GARCH models (ARCH, GARCH, EGARCH and GJR-GARCH models) in four of Bucharest Stock Exchange indices which reflect only the evolution of market prices: Bucharest Exchange Trading Index, Bucharest Exchange Trading Extended Index, Bucharest Exchange Trading – Investment Funds and Bucharest Exchange Trading Energy & Related. The empirical results reveal in three cases out of four that volatility turned out to react asymmetrically to good and bad news. Thereby, one more time EGARCH model turned out as being both the best and the predominant model in estimating the conditional variance of financial time series. Since the conditional variance is time varying, Petrică and Stancu (2017b) devoted time and attention in acquiring some conclusions in case of the exchange rate. Thus, they went from indices to analyze the changes in volatility in case of the EUR/RON exchange rate using different specifications of GARCH models (ARCH, GARCH, EGARCH, TARCH and PARCH). The EGARCH and PARCH models perform well, but the best model for estimating daily returns of the EUR/RON exchange rate was again EGARCH (EGARCH(2,1) with asymmetric order 2 under the assumption of Student's t distributed innovation terms). As can be seen, our recent papers have focused on the study of the changes in volatility in only one market (financial or monetary) and we found that EGARCH and PARCH models are quite close. This paper comes now to show what happens if we are using those two asymmetric GARCH models but in different stock markets, a developed one and an underdeveloped stock market. Thus, the aim is to compare the EGARCH and PARCH models in markets at opposite poles: London Stock Exchange represented by FTSE 100 Index (developed stock market) and Bulgarian Stock Exchange represented by SOFIX Index (underdeveloped stock market).

Methodology

The mathematical representation of an asymmetric GARCH model implies two equations: conditional mean and conditional variance, that have to be estimated simultaneous. According to Petrică, Stancu and Tindeche (2016, p.11) the conditional mean has the following representation:

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_r y_{t-r} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \cdots + \beta_s \varepsilon_{t-s} \quad (1)$$

where:

y_t, y_{t-1}, y_{t-2}, ..., y_{t-r} - the realisation of the dependent variable Y at time $t, t-1, \dots, t-r$;

α_1, α_2, ..., α_r, β_1, β_2, ..., β_s - the unknown parameters of the model, $\alpha_1 \neq 0$, $\beta_s \neq 0$;

ε_t - the value of the disturbance term at time t, i.i.d. $\varepsilon_t \sim N(0, \sigma^2)$;

r - the number of lagged values of Y and represents the order of the autoregressive process;

$\varepsilon_{t-1}, \varepsilon_{t-2}$, ..., ε_{t-s} - the realisation of the lagged disturbances;

s - the number of lagged disturbances and represents the order of the moving average process.

* Normal distribution, Student's t distribution, Generalized Error distribution (GED), Student's t distribution with fixed degrees of freedom, and GED distribution with fixed parameter.

After determining the adequate order of the parameters r and s, next step consists in estimating the parameters α_1, α_2, ..., α_r, β_1, β_2,..., β_s of equation (1) and then calculate and estimate EGARCH and PARCH models on $\hat{\varepsilon}_t$.

The Exponential Generalized Autoregressive Conditionally Heteroscedastic Model (EGARCH)

Rachev, et al. (2007, p.301) state "the asymmetric behavior of asset returns is modeled as an asymmetric, nonlinear specifications of the conditional variance process and a symmetric distribution (such as Gaussian or the Student's t-distribution) for the conditional error" and present the EGARCH(p,q) model, introduced by Nelson (1991), as follows:

$$\log(h_t) = a_0 + \sum_{i=1}^{q} a_i g(\eta_{t-i}) + \sum_{i=1}^{p} b_i \log(h_{t-i}) \tag{2}$$

where:

h_t – the conditional variance of the disturbances at time t;

a_0 – the constant term;

$\varepsilon_t = \sqrt{h_t}\eta_t$ and $g(\eta_t) = \theta\eta_t + \gamma[|\eta_t| - E|\eta_t|]$ – the weighted disturbances that model asymmetric effects between positive and negative asset returns with θ, γ – constants.

The Power Autoregressive Conditionally Heteroscedastic Model (PARCH)

The PARCH model, introduced by Ding, et al. (1993), may be specified as follows:

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{q} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \cdot \varepsilon_{t-i})^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta} \tag{3}$$

where:

ω – the constant term, with $\omega > 0$;

α_i, β_j – the standard ARCH and GARCH coefficients with $\alpha_i \geq 0$ and at least one $\alpha_i > 0$, $i = \overline{1,q}$, and $\beta_j \geq 0$, $j = \overline{1,p}$;

γ_i – the asymmetry coefficients ($|\gamma_i| < 1$) and δ – the coefficient for the power term ($\delta > 0$).

Empirical Results and Discussion

In this section we employ EGARCH and PARCH models to the percentage daily returns of the FTSE 100 and SOFIX Indices ($Return_t = \log\left(\frac{Closing\_Price_t}{Closing\_Price_{t-1}}\right) * 100$), in order to model the conditional volatility in London Stock Exchange and Bulgarian Stock Exchange. The data is acquired directly from the Bloomberg database and concerns the period January 04, 2010 to September 27, 2016. The FTSE UK Index series highlights the performance of U.K. companies and affords investors "a comprehensive and complementary set of indices that measure the performance of all capital and industry segments of the UK equity market"[†]. (Table no. 1) provides some basic statistics of the FTSE 100 Index series, while (table no. 2) shows the non-stationarity of the series using Augmented Dickey–Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) unit root tests:

[†] http://www.londonstockexchange.com/statistics/ftse/ftse.htm

Table no. 1: Basic statistics of daily FTSE 100 Index (January 04, 2010 to September 27, 2016)

Basic statistics			
Mean	7542.548	Skewness	0.329820
Std. Dev.	964.2274	Kurtosis	2.472135

Source: Authors' computations

Table no. 2: Unit root tests (with constant term and time trend) on daily FTSE 100 Index (ADF, PP and KPSS)

Unit Root Test	Calculated value	Critical value 1%	Critical value 5%	Critical value 10%
ADF	-2.814556 (0.1921)	-3.963457	-3.412458	-3.128178
PP	-2.603504 (0.2788)	-3.963444	-3.412451	-3.128174
KPSS	0.379370	0.216000	0.146000	0.119000

Source: Authors' computations

"Volatility, a symptom of market disruption, is associated with unpredictability, uncertainty and is usually realized through time varying conditional variance."[‡] (Table no. 3) shows that in case of percentage returns we get stationarity:

Table no. 3: ADF unit root test (with constant term and time trend) on FTSE 100 daily returns

Unit Root Test	Calculated value	Critical value 1%	Critical value 5%	Critical value 10%
ADF	-22.95322 (0.0000)	-3.963457	-3.412458	-3.128178

Source: Authors' computations

Forwards, we are using the Box-Jenkins methodology in order to come up with the adequate ARMA model for the conditional mean. After considering a number of specifications, we selected ARMA (1,4) model based on minimum Akaike Information Criterion and Hannan-Quinn Criterion:

$$\hat{R}_t^{FTSE100} = 0.595756\,\hat{R}_{t-1}^{FTSE100} + \varepsilon_t - 0.524742\,\varepsilon_{t-1} - 0.108079\,\varepsilon_{t-2} + 0.061371\,\varepsilon_{t} - 0.126340\,\varepsilon_{t-4} \quad (4)$$

and testing the residuals from equation (4) for ARCH effects we get that we can run the asymmetric GARCH models (table no. 4):

Table no. 4: EViews 9 output of the ARCH LM Test

Obs*R-squared	138.3361	Prob. Chi-Square(1)	0.0000

Source: Authors' computations

Therefore, applying different specifications of EGARCH and PARCH models, (table no. 5) reports EGARCH(1,2) model under Student's t distribution as being the best model, while (table no. 6) reports the PARCH(1,1) model under Student's t distribution.

[‡] Sabiruzzaman, et al. (2010, p.142).

Table no. 5: Estimation results of EGARCH model for the FTSE 100 Index

Variable	EGARCH Asymmetric order 1							
	EGARCH(1,1)		EGARCH(1,2)		EGARCH(2,1)		EGARCH(2,2)	
	Normal	Student's t	Normal	Student's t	Normal	Student's t	Normal	Student's t
Variance Equation								
C(7)	-0.14213 (0.0000)	-0.14968 (0.0000)	-0.19786 (0.0000)	-0.19013 (0.0000)	-0.12954 (0.0000)	-0.13738 (0.0000)	-0.18984 (0.0000)	-0.18018 (0.0001)
C(8)	0.18849 (0.0000)	0.19381 (0.0000)	0.26132 (0.0000)	0.24598 (0.0000)	0.25942 (0.0000)	0.25842 (0.0001)	0.27243 (0.0000)	0.26400 (0.0000)
C(9)	-0.11847 (0.0000)	-0.18009 (0.0000)	-0.15822 (0.0000)	-0.21894 (0.0000)	-0.08781* (0.1416)	-0.08089* (0.2646)	-0.0217* (0.7699)	-0.0310* (0.7194)
C(10)	0.95007 (0.0000)	0.93351 (0.0000)	0.46603 (0.0002)	0.58302 (0.0003)	-0.11505 (0.0000)	-0.17493 (0.0000)	-0.15584 (0.0000)	-0.2149 (0.0000)
C(11)			0.47053 (0.0002)	0.34156 (0.0274)	0.95528 (0.0000)	0.94085 (0.0000)	0.49871 (0.0073)	0.61602 (0.0026)
C(12)							0.44015 (0.0138)	0.31191* (0.1099)
AIC	2.84447	2.79809	2.84129	2.79679	2.84459	2.79846	2.84241	2.79786
* The coefficient is not significant at any confidence level (1%, 5% and 10%).								

Source: Authors' computations

Table no. 6: Estimation results of PARCH model for the FTSE 100 Index

Variable	PARCH Asymmetric order 1							
	PARCH(1,1)		PARCH(1,2)		PARCH(2,1)		PARCH(2,2)	
	Normal	Student's t	Normal	Student's t	Normal	Student's t	Normal	Student's t
Variance Equation								
C(7)	0.05248 (0.0000)	0.07352 (0.0000)	0.07031 (0.0000)	0.08673 (0.0000)	0.04803 (0.0000)	0.07135 (0.0000)	0.08273 (0.0000)	0.08344 (0.0000)
C(8)	0.09643 (0.0000)	0.10705 (0.0000)	0.13876 (0.0000)	0.13027 (0.0000)	0.12927 (0.0001)	0.11248 (0.0000)	0.08786 (0.0000)	0.13793 (0.0132)
C(9)	0.63205 (0.0000)	0.99999 (0.0000)	0.64937 (0.0000)	0.99989 (0.0000)	0.46959 (0.0011)	0.99990 (0.0000)	0.99982 (0.0000)	0.94221 (0.0409)
C(10)	0.86544 (0.0000)	0.83921 (0.0000)	0.31589 (0.0145)	0.39366 (0.0126)	-0.03676* (0.3327)	-0.02659* (0.2210)	0.04373 (0.0150)	-0.02012 (0.6094)
C(11)	1.37819 (0.0000)	1.14197 (0.0000)	0.50096 (0.0000)	0.40117 (0.0051)	0.87909 (0.0000)	0.85670 (0.0000)	0.18071 (0.0430)	0.42603 (0.0111)
C(12)			1.25454 (0.0000)	1.26047 (0.0000)	1.23795 (0.0000)	1.15637 (0.0000)	0.59524 (0.0000)	0.38331 (0.0097)
C(13)							1.56909 (0.0000)	1.26685 (0.0000)
AIC	2.84215	2.79206	2.83827	2.79293	2.84306	2.79217	2.83919	2.79381
* The coefficient is not significant at any confidence level (1%, 5% and 10%).								

Source: Authors' computations

Hence, (table no. 5) and (table no. 6) reveal together the presence of the leverage effect through EGARCH and PARCH models. Moreover, the Student's t distribution of the innovations performs better than the Gaussian distribution. Taking these into account and based on the Akaike Information Criterion we find that PARCH is the most adequate model.

Analogously to the FTSE 100 Index, the first index developed by the Bulgarian Stock Exchange is given by the SOFIX Index and represents "a correlation of the sum of the market capitalization of the companies within the index portfolio on the current day and the sum of the market capitalization of the same on the previous day"[§]. Next, (table no. 7) provides that the hypothesis of a unit root cannot be rejected, while (table no. 8) indicates stationarity in case of the transformed series:

Table no. 7: Unit root tests (with constant term and time trend) on daily SOFIX Index

Unit Root Test	Calculated value	Critical value		
		1%	5%	10%
ADF	-1.473619 (0.8384)	-3.963571	-3.412514	-3.128211
PP	-1.515612 (0.8242)	-3.963568	-3.412512	-3.128210
KPSS	0.490336	0.216000	0.146000	0.119000

Source: Authors' computations

Table no. 8: ADF unit root test (with constant term and time trend) on daily returns SOFIX Index

Unit Root Test	Calculated value	Critical value		
		1%	5%	10%
ADF	-38.01316 (0.0000)	-3.963571	-3.412514	-3.128211

Source: Authors' computations

Forwards, using the Box-Jenkins methodology we find that the adequate model for the conditional mean is AR(1) model having the following equation:

$$\hat{R}_t^{SOFIX} = 0.068762\hat{R}_{t-1}^{SOFIX} + \varepsilon_t \quad (5)$$

and according to ARCH LM Test (table no. 9) the conditional variance is time varying:

Table no. 9: EViews 9 output of the ARCH LM Test

Obs*R-squared	94.92900	Prob. Chi-Square(1)	0.0000

Source: Authors' computations

Whatever the error term distribution, Gaussian or Student's t, we observe that the asymmetry coefficient (γ) is not significant in any EGARCH or PARCH models (table no. 10) and (table no. 11). Thereby, both EGARCH and PARCH are not adequate models for estimating the conditional variance of SOFIX Index.

[§] https://en.wikipedia.org/wiki/SOFIX

Table no. 10: Estimation results of EGARCH model for the SOFIX Index

Variable	EGARCH Asymmetric order 1							
	EGARCH(1,1)		EGARCH(1,2)		EGARCH(2,1)		EGARCH(2,2)	
	Normal	Student's t	Normal	Student's t	Normal	Student's t	Normal	Student's t
Variance Equation								
C(3)	-0.28047 (0.0000)	-0.32946 (0.0000)	-0.28134 (0.0000)	-0.34441 (0.0000)	-0.27908 (0.0000)	-0.31045 (0.0000)	-0.19861* (0.4479)	-0.20074* (0.5403)
C(4)	0.31003 (0.0000)	0.37036 (0.0000)	0.31104 (0.0000)	0.38833 (0.0000)	0.31297 (0.0000)	0.39672 (0.0000)	0.32312 (0.0000)	0.40679 (0.0000)
C(5)	-0.02265* (0.1066)	-0.00840* (0.7424)	-0.02272* (0.1306)	-0.00796* (0.7675)	-0.00434* (0.9180)	-0.04611* (0.5173)	-0.10310* (0.7210)	-0.17921* (0.6189)
C(6)	0.90148 (0.0000)	0.88749 (0.0000)	0.89686 (0.0000)	0.80823 (0.0000)	-0.02265* (0.1104)	-0.00602* (0.8084)	-0.01718* (0.4661)	0.00017* (0.9924)
C(7)			0.00438* (0.9714)	0.07611* (0.6466)	0.90216 (0.0000)	0.89670 (0.0000)	1.17818* (0.2069)	1.21942* (0.2300)
C(8)							-0.24722* 0.7690	-0.28493* (0.7531)
AIC	2.34146	2.27882	2.34266	2.27984	2.34266	2.27977	2.34378	2.28078
* The coefficient is not significant at any confidence level (1%, 5% and 10%).								

Source: Authors' computations

Table no. 11: Estimation results of PARCH model for the SOFIX Index

Variable	PARCH Asymmetric order 1							
	PARCH(1,1)		PARCH(1,2)		PARCH(2,1)		PARCH(2,2)	
	Normal	Student's t	Normal	Student's t	Normal	Student's t	Normal	Student's t
Variance Equation								
C(3)	0.05702 (0.0000)	0.07227 (0.0001)	0.05474 (0.0001)	0.07547 (0.0006)	0.05809 (0.0000)	0.06640 (0.0004)	0.07656 (0.0060)	0.00864 (0.2958)
C(4)	0.13776 (0.0000)	0.19328 (0.0000)	0.13236 (0.0003)	0.20226 (0.0003)	0.12543 (0.0006)	0.21107 (0.0003)	0.11227 (0.0013)	0.18384 (0.0000)
C(5)	0.02340* (0.4758)	0.00266* (0.9615)	0.02233* (0.4958)	0.00268* (0.9614)	0.02550* (0.4915)	-0.00321* (0.9458)	0.02885* (0.5857)	-0.00977* (0.4046)
C(6)	0.64315 (0.0000)	0.66874 (0.0000)	0.68561 (0.0007)	0.59966 (0.0151)	0.01510* (0.5893)	-0.03208* (0.5578)	0.07607* (0.1991)	-0.16206 (0.0005)
C(7)	3.37319 (0.0000)	2.47335 (0.0000)	-0.03149* (0.8180)	0.05451* (0.7659)	0.62785 (0.0000)	0.69356 (0.0000)	0.26237* (0.5944)	1.57717 (0.0000)
C(8)			3.39618 (0.0000)	2.46593 (0.0000)	3.43046 (0.0000)	2.47450 (0.0000)	0.21994* (0.4798)	-0.61468 (0.0000)
C(9)							3.54601 (0.0000)	2.42523 (0.0000)
AIC	2.32781	2.27491	2.32897	2.27602	2.32890	2.27593	2.32967	2.27565
* The coefficient is not significant at any confidence level (1%, 5% and 10%).								

Source: Authors' computations

Conclusion

This paper comes up with a new approach which consists in employing the most predominant asymmetric GARCH models through comparison (EGARCH and PARCH), but in two stock markets at opposite poles: a developed stock market represented by the London Stock Exchange and an underdeveloped stock market – the Bulgarian Stock Exchange. The empirical results provide that the developed market is characterized by leverage effect (the PARCH(1,1) with asymmetric order 1 and Student's t distribution is the most adequate model for estimating the conditional variance in case of the FTSE 100 Index), but the interesting point in the paper is given by what is happening in case of the underdeveloped market, where neither EGARCH nor PARCH are good models for estimating the conditional variance of SOFIX Index. Future research should consist in the underdeveloped market analysis using symmetric GARCH models.

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